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Maximum Likelihood Indoor Localization of a WiFi Radio Transmitter with Structural Knowledge

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ABSTRACT

In this paper, we present a method for estimating the location of a WiFi transmitter by a receiver using the radio resource and knowledge of the indoor room structure. We derive a three-ray path propagation model for the received radio signal in a known indoor environment. We show that the position of the transmitter could be localized using the received radio signal measurements. The likelihood under this model exhibits multiple local peaks when only few frequencies are used, which leads to the location ambiguities under the Maximum Likelihood criterion. We observed in simulation that the ambiguous locations under the MLE vary with the WiFi radio frequency used but the ground truth location is always presented as a peak. Therefore, we use multiple WiFi frequency bands to resolve the localization ambiguity. A subspace based method is applied in combination with MLE utilizing the same measurements to improve localization efficiency. Simulation using commercial ray tracing software presents promising result.

Keywords
Maximum Likelihood, location ambiguity, WiFi frequency, subspace

1. INTRODUCTION

WiFi-based indoor localization has drawn increasing attention in recent years, due to its low cost, ease of implementation and pervasive practical application. Traditional WiFi-based localization methods typically resort to signal parameters such as time of flight based approach for range estimation and then localize the target of interest by making use of multiple access points and triangulation technique [1], or utilize powerful receiver structure such as antenna array for angle of arrival identification and angulation. However, the accuracy and stability of the aforementioned methods usually are affected by multipath interference in indoor environment. On the other hand, even though multipath fading effects can lead to non-neglectable signal variance, i.e., constructive and destructive fading, with respect to different operation frequencies and transceiver locations [2], this channel frequency selective fading property may turn out to be helpful in providing an unique mapping for localization purpose.

As WiFi can operate at both 2.4GHz and 5GHz band, we can exploit its multipath interference and frequency selective fading property to localize the WiFi transmitter. We consider a WiFi based localization scenario which involves only one radio transmitter and receiver pair but with known room structure and in the presence of direct path signal. A three-ray path propagation model is derived and a Maximum Likelihood estimation (MLE) based approach is proposed to estimate the transmitter location using the available received WiFi signal, which is a complex number in general. To remove location ambiguity, measurements across all the available WiFi frequency bands may be utilized. The use of multiple bands for localization has appeared in recent literature. For example, in [3], WiFi signals at 2.4 GHz and 5 GHz spanning almost one GHz of bandwidth with a total number of 35 WiFi center frequency bands are used for time of flight estimation. As indicated in [4], the multiple bands WiFi measurements (complex number) can be conveniently extracted from Channel State Information (CSI) in Intel 5300 cards.

This paper is organized as follows. Following the Introduction, we first formulate the receive signal data model in Section II. Then we describe the location ambiguity issue in MLE due to multipath signal interference and briefly explain the idea of addressing ambiguity problem by utilizing multiple WiFi frequency bands in Section III. In Section IV, based on the measured multiple frequency signal, MLE is derived, together with a subspace based method to improve estimation efficiency. In Section V, simulation of MLE localization result using commercial software for scene setting and measurement generation is demonstrated. Finally, conclusions are given in Section VI.

2. THE DATA MODEL

Consider the problem of localizing a radio transmitter in a two dimensional space with known room structure and a fixed receiver whose location is given, as shown in Fig. 1. A radio frequency signal is transmitted by the transmitter and multiple copies of the same signal with different time delay and attenuation is superpositioned at the receiver due to reflection. In order to account for this multipath interference as well as reduce complexity, reflected signals that only bounce less than two times are taken as effective components
in the receiving signal, owing to their relatively significant decaying of amplitude during reflection, while the rest of which are regarded as additional noise.

According to [5], the channel response for frequency \( f_i \) given the transmitter location \( \varphi = (x, y) \) can be modeled as

\[
h_i(\varphi) = \sum_{p=0}^{P} a_p(\varphi) \exp^{-j2\pi f_i \tau_p(\varphi)}, \quad i = 1, 2, \cdots N, \tag{1}
\]

where \( a_p(\varphi) \) and \( \tau_p(\varphi) \) are the amplitude and time delay for the \( p^{th} \) path, with \( p = 0 \) representing the direct path. Recall that \( h_i(\varphi) \) is provided as CSI in some available commercial chips, based on the 802.11 standard, such as Intel 5300 NIC [4].

As the value of \( a_p \) and \( \tau_p \) are both functions of the transmitter location \( \varphi \), the channel response \( h_i \) is directly dependent on the transmitter location.

To model the change of amplitude with respect to the transmitter location, we use free space path-loss model and add a reflection decaying factor to account for the additional power loss during reflection. Note that \( a_p(\varphi) \) represents the attenuation in the electrical field of the receiving signal. Therefore, \( a_p(\varphi) \) can be expressed as

\[
a_p(\varphi) = \frac{r_p}{d_p(\varphi)}, \tag{2}
\]

where \( d_p(\varphi) \) represents the \( p^{th} \) path travelling distance and \( r_p \) is the reflection delaying factor for the \( p^{th} \) path. Obviously, there is no reflection delaying for direct path, i.e. \( r_0 = 1 \). Because in the free space path-loss model, the receiving power is inversely proportional to the distance square, the electrical magnitude is therefore inversely related to distance, as shown in (2).

The received signal \( \tilde{h}_i \) for frequency \( f_i \) given the transmitter location is modeled as a noisy complex value, i.e.

\[
\tilde{h}_i(\varphi) = h_i(\varphi) + w_i, \quad |w_i| \sim N(0, \sigma_i^2), \tag{3}
\]

where \( w_i \) represents the noise and is assumed as complex Gaussian distributed, i.e., the magnitude of \( w_i \) is zero mean Gaussian distribution and the phase of \( w_i \) is uniform distributed.

Hence, the probability density function of a measurement \( \tilde{h}_i \) given \( \varphi \) as the location of transmitter is

\[
p(\tilde{h}_i; \varphi) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp \left\{ -\frac{1}{2\sigma_i^2} (\tilde{h}_i - h_i(\varphi))(\tilde{h}_i - h_i(\varphi))^* \right\}. \tag{4}
\]

3. LOCATION AMBIGUITY ISSUE

Due to the constructive and deconstructive interference of multipath signals, the likelihood function exhibits multiple local maximums with respect to different transmitter locations. Therefore, MLE based localization approach is very sensitive to noise, as likelihood magnitudes of several local maximums are quite close to each other for a single frequency, leading to multiple unresolvable candidate transmitter locations, i.e., location ambiguities.

![Figure 1](image1.png)

**Figure 1:** Considered three ray propagation path model: a fixed receiver (Rx) whose location is known receives three components of superpositioned sinusoids emitted by a radio transmitter (Tx) with unknown location in a given 5m × 5m room. Each signal path undergoes different attenuations and time delays.

![Figure 2](image2.png)

**Figure 2:** Likelihood map demonstrating ambiguous transmitter locations when a single frequency is used.

To illustrate this, consider the scenario in Fig. 1 where the true transmitter’s location is at (1,1) and the likelihood value is evaluated around its true location, with a step size of 2 centimeters. As (it) can be seen in Fig. 2, multiple locations have very close likelihood value compared with the true location, leading to unresolvable ambiguities.

In order to tackle the ambiguity issue, we leverage the fact that the WiFi signals span over multiple frequency bands and naturally exploit the diversity of the receiving signal for different frequencies. The key idea is that for each individual frequency, even though its corresponding ambitious location during likelihood evaluation varies, the true location shall always contributes to a local maximum. Therefore, by combining multiple frequency measurements, transmitter location ambiguity can be possibly removed. See Fig. 3 for an explanation. It can be seen that the signal to noise ratio is dramatically improved compared to the single frequency result in Fig. 2. Hence location ambiguity could be greatly
mitigated by exploiting WiFi signal at multiple frequency bands.

Figure 3: Remove transmitter location ambiguity by using multiple frequencies: as shown by the likelihood map, the true location at (1,1) contributes to the largest likelihood compared to its neighbouring locations.

To further evaluate the ability of ambiguity suppressing by applying multiple WiFi frequency signal, we calculate the Fisher information matrix \( I(\phi) \) as [6]

\[
[I(\phi)]_{m,n} = -E \left[ \sum_{i=1}^{N} \frac{\partial^2}{\partial \phi_m \partial \phi_n} \log p(\tilde{h}_i; \phi) \right],
\]

(5)

where \( N \) is the total number of frequencies and \( E[\cdot] \) represents expectation operator with respect to the measurements.

Figure 4: Determinant of Fisher information matrix using 35 frequencies.

To evaluate how much information we can get by applying MLE, we compute \( I(\phi) \) with respect to all possible transmitter locations using (5) and take the corresponding determinant. Fig. 4 demonstrates the determinant value of Fisher information matrix when the receiver is placed at (2.5, 5). In this special case, Fisher information exhibits symmetry property as the receiver is positioned in symmetry with the room structure. No information can be acquired along the vertical line at \( x = 2.5m \) as Fisher information matrix is singular.

Remarks:

- Although it is evident employing multiple frequencies can help remove location ambiguity, the minimum number of the required frequency along with how to select them among the available frequency bands in an optimal sense remains to be an ongoing research. Obviously, the more frequency employed, the less ambiguity can be achieved. Based on simulation result, frequencies selected across 2.4GHz band and 5GHz band can provide better ambiguity suppressing performance than those selected from either the 2.4GHz band or the 5GHz band only.

- Due to the fact that high frequency WiFi signal has a short wavelength (centimeter level), the likelihood function changes over distance on the order of several centimeters. Therefore how to identify the global maximum can be very challenging in practice. Similar problem has also been mentioned in [7] using MLE for parameter estimation. We propose to meet the computation requirement for global maximum identification in combination with a subspace approach for time delay offset estimation, using the same set of measurements, which will be discussed in the Section IV.

4. TRANSMITTER LOCALIZATION

In this section, we turn to the practical procedures on how to employ MLE to find the radio transmitter location. First the maximum likelihood function is reformulated with respect to multiple WiFi frequency and multiple samples for each frequency. Second, in order to reduce the computation time, a subspace based approach is used in combination with MLE to improve computation efficiency.

4.1 Maximum Likelihood Approach

As discussed above, transmitter location can’t be identified by MLE with a single frequency due to ambiguity. Hence, we utilize multiple frequencies in CSI and formulate the location problem as follows: denote the \( k \)th measurement set of all the available \( N \) frequencies as \( \tilde{h}_k = [\tilde{h}_{1k}, \tilde{h}_{2k}, \cdots, \tilde{h}_{Nk}]^T \), whose distribution is modeled as joint Gaussian

\[
p(\tilde{h}_k; \phi) = \exp \left\{ -\frac{1}{2} (\tilde{h}_k - h_k(\phi))^\ast \Sigma_k^{-1} (\tilde{h}_k - h_k(\phi)) \right\} ,
\]

(6)

where\( (\cdot)^T \) and \( (\cdot)^\ast \) represents transpose and Hermitian conjugate transpose respectively, and \( \Sigma_k = \sigma^2 I_N \) is the covariance matrix of noise.

In practice, for all the available frequencies, we might be able to take multiple measurement samples. Assume \( K \) samples are taken for each frequency. Then all the measurements can be stacked as a \( N \times K \) measurement matrix \( \tilde{H} = [\tilde{h}_1, \tilde{h}_2, \cdots, \tilde{h}_K] \). Therefore, drop the constant value, the log likelihood of \( \tilde{H} \) can be written as

\[
\log p(\tilde{H}; \phi) = \sum_{k=1}^{K} -\frac{1}{2} (\tilde{H}_k - H_k(\phi))^\ast \Sigma_k^{-1} (\tilde{H}_k - H_k(\phi)).
\]

(7)
The MLE criterion is to find a location point \( \varphi = (x, y) \)
that leads to the largest likelihood value. It is worth noting
that under the Gaussian white noise assumption, MLE is
equivalent to a nonlinear least-square error minimization
problem.

4.2 Path Delay Offset Estimation

To find the highest peak in the global sense can be ex-
tremely time consuming in general, since the distance be-
tween any two closest local maxima is in the order of half
the wavelength, which is about six centimeters for 2.4GHz
WiFi signal. The magnitude of likelihood function varies
rapidly with respect to the transmitter location due to the
interference between multipath WiFi signal components,
which have relatively short wavelengths. In other words,
a slight change of the transmitter location can make a signifi-
cant change in the corresponding receiving signal. This will
pose a challenge for the MLE global maximum identification.

To improve computation efficiency, we further utilize CSI
and exploit time delay information contained therein. We
rewrite the N frequency channel response in (3) by a matrix
form,

\[
\begin{bmatrix}
\tilde{h}_1 \\
\tilde{h}_2 \\
\vdots \\
\tilde{h}_N
\end{bmatrix} = [F(\tau_1) \ F(\tau_2) \ \cdots \ F(\tau_p)]
\begin{bmatrix}
a_1 \\
a_2 \\
\vdots \\
a_p
\end{bmatrix} + \begin{bmatrix}
w_1 \\
w_2 \\
\vdots \\
w_N
\end{bmatrix}
\]

(8)

with \( F(\tau_p) = [\exp^{-j2\pi f_1 \tau_p}, \exp^{-j2\pi f_2 \tau_p}, \cdots, \exp^{-j2\pi f_N \tau_p}]^T. \)

Therefore, \( F(\tau_p) \) can be regarded as a steering vector
containing the unknown parameter of the \( p^{th} \) path time delay
\( \tau_p \). Without resorting to any other hardware or precise
measurements, a subspace based approach such as [8],[9]
can be employed for path delay estimation using the same

5. SIMULATION

In this simulation, we consider a two dimensional space
with known structure and three main path signal compo-
nents in the receiving signal. As shown in Fig. 6, the re-
ciever’s location is given as (3, 5). A transmitter grid set
with a large number of transmitters spaced by 5 centimeters
between each other is placed to evaluate how the signal
changes with location. For the sake of clear illustration,
we randomly pick up one of the transmitter among the grid
and demonstrate its three main signal propagation paths
that are considered in the model.
The ray tracing software [10] can provide the receiving signal power (in dBm) and phase (in degree) with respect to different transmitter location. To simulate CSI data, a postprocessing of the software output is needed to convert the receiving signal into the complex form as shown in (1). This can be done either by making use of the complex impulse response output or the receiving power output. Then a Gaussian complex noise is added to each measurement to account for the measurement uncertainty in CSI.

In order to evaluate the effectiveness of the MLE approach for radio transmitter location estimation, we randomly pick up a transmitter (location coordinate is (1.53, 0.96)) in the transmitter grid set in Fig. 6 and extract its corresponding signal measurements of all the available frequencies for MLE based localization.

When we make use of all the available 35 frequencies spanning from 2.4GHz to 5GHz band, ambiguity can be removed and the true target location can be identified. Fig. 7 shows the result for one simulation. As the subspace method can provide with transmitter’s location area (rectangular drew by dashed line), efficiency can be improved during the MLE procedure as multiple local maximum location outside of this location area can be avoided for likelihood evaluation. This demonstrates that the MLE is able to find the global maximum and hence localize the transmitter.

![Simulation result using 35 frequencies](image)

**Figure 7: Simulation result using 35 frequencies**

Please note that the subspace method can only help to narrow down the searching area for the global maximum point in MLE estimation. To account for uncertainty and avoid risky decision making for transmitter location, a relatively large searching area needs to be defined. So there is a tradeoff in practice. A more sophisticated approach should be developed for global maximum identification in the future.

6. CONCLUSION

The WiFi based radio transmitter localization using maximum likelihood approach contributes in the following ways. First, instead of using multiple receivers, only one receiver is employed for transmitter location estimation, which is more practical in today’s mobile application and is immune from multiple receivers deployment difficulty. Second, we naturally exploit the CSI which is available in commercial chips and make use of multiple WiFi frequencies to remove location ambiguity. Third, by making use of the same set of measurements, a subspace based method is utilized for transmitter location area estimation, which can be combined with MLE to improve localization efficiently. Future work will focus on radio transmitter localization in the unknown and time variant environment.

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